THE INFLUENCE OF CONSTANT ELECTRIC AND MAGNETIC FIELDS ON THE SPIN OF THE PARTICLE

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ABSTRACT. If we assume that the particle obeys the second order equation obtained in the usual way of operating twice the Dirac's linear Hamiltonian, we find that the change of momentum and spin direction of a particle subjected to magnetic and electric fields is the same as for a particle obeying Dirac's first order equation

INTRODUCTION

Tolhoek (1951) has discussed the change of momentum and spin orientation of an electron beam when subjected to constant transverse and longitudinal electric and magnetic fields. He has treated the problem for the electron according to three methods, viz , Klein-Gordon's equation, Pauli spin theory and Dirac's linear equation. The result for the change of momentum direction obtained from the Klein-Gordon equation is the same as one gets from the Dirac's equation, whereas the result from the Pauli spin theory is the non-relativistic approximation of the above For the change of spin orientation what the Pauli spin theory gives is again the non-relativistic limit of what is obtained from Dirac's theory.

If we operate twice the Dirac linear Hamiltonian, we obtain the following equation ·

$$\left[(E-e\phi)^2-(c\boldsymbol{p}-c\boldsymbol{A})^2-m^2c^4+e\hbar c\sigma \,.\, \boldsymbol{H}+iehc\alpha.\ \vec{c}\right]\psi_u = 0 \qquad \ldots \quad (1)$$

Let us consider the influence of transverse electric field along y-axis on the orientation of spin direction of the electron moving along x-axis and obeying the above equation.

We have

$$\mathcal{E} = \epsilon \boldsymbol{j}, \quad \phi = \epsilon y, \quad \boldsymbol{A} = 0 \qquad \ldots \quad (2)$$

As before we put

$$\psi = e^{ief(x,y)}\,\psi_0 \qquad \ldots \quad (3)$$

where ψ_0 is the usual solution of the Dirac's equation for a free electron

$$\psi_0 = \begin{pmatrix} -\dfrac{cp_zA}{E+mc^2} - \dfrac{c(p_x-ip_y)B}{E+mc^2} \\ -\dfrac{c(p_x+ip_y)A}{E+mc^2} + \dfrac{cp_zB}{E+mc^2} \\ A \\ B \end{pmatrix} e^{\frac{i}{\hbar}(\mathbf{px}-et)} = \begin{pmatrix} -\dfrac{cpB}{E+mc^2} \\ -\dfrac{cpA}{E+mc^2} \\ A \\ B \end{pmatrix} e^{\frac{i}{\hbar}(px-Et)} \quad \ldots (4)$$

Since $p_y = p_z = 0$ and $p_x = p$. Under the conditions of (2), equation (1) becomes

$$[E^2-m^2c^4-c^2p_x^{\,2}-c^2p_y^{\,2}-c^2p_z^{\,2}+2eE\epsilon y+ie\hbar c\alpha_y\epsilon]\,\psi = 0 \quad \ldots (5)$$

Substituting (2) into (5), we have

$$\partial f/\partial x - \frac{i\hbar}{2p}(\partial^2 f/\partial x^2+\partial^2 f/\partial y^2+\partial^2 f/\partial z^2) - \frac{E\epsilon y}{c^2\hbar p} - \frac{\iota\epsilon\alpha_y}{2p} = 0 \quad \ldots (6)$$

Equation (6) can be satisfied by the following relation

$$f = (\epsilon E/\hbar pc^2)xy + i(\epsilon/2pc)\alpha_y \quad \ldots (7)$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi \end{pmatrix} = e^{ie}[(\epsilon E/\hbar pc^2)xy]\left[1 + \frac{ie\epsilon}{2pc}x\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}\right]\begin{pmatrix} -\dfrac{cpB}{E+mc^2} \\ -\dfrac{cpA}{E+mc^2} \\ A \\ B \end{pmatrix} e^{\frac{i}{\hbar}(px-Et)} \quad \ldots (8)$$

We can write

$$\begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} K(x,y,t) + \frac{ie\epsilon}{2pc}x\,\frac{cp}{E+mc^2}\begin{pmatrix} -A \\ B \end{pmatrix} K(x,y,t) \quad . (9)$$

$$= \left(1 - \frac{i}{2}\sigma_z\,\frac{e\epsilon}{E+mc^2}\,x\right)\begin{pmatrix} A \\ B \end{pmatrix} K(x,y,t) \quad .. (11)$$

where $K(x, y, t) = e^{(i/\hbar)(px-Et)+ie(\epsilon E/hpc^2)_{xy}}$

According to Darwin (1928)

$$-B/A = \cot\frac{\chi}{2}e^{i\omega} \qquad \ldots \quad (12)$$

where χ and ω are colatitude and longitude of electron spin respectively From (10)

$$-\frac{\psi_4}{\psi_3} = -\frac{B}{A}\cdot\frac{1+\dfrac{iecx}{2(E+mc^2)}}{1-\dfrac{ie\epsilon x}{2(E+mc^2)}} \approx \cot\tfrac{1}{2}\chi\, e^{i\omega}\,.\, e^{i\frac{e\epsilon}{E+mc^2}x} \qquad \ldots \quad (13)$$

Therefore we obtain for the rotation of spin axis about z-axis

$$\Delta\alpha_{\epsilon 1} = [e\,\epsilon/(E+mc^2)]x \qquad \ldots \quad (14)$$

Also

$$\Delta p_x = 0, \quad \Delta p_z = 0, \quad \Delta p_y = (e\,\epsilon E/pc^2)x, \quad \Delta\gamma_\epsilon = \frac{\Delta p_y}{p_z} = \frac{eE\epsilon}{p^2c^2}x \qquad \ldots \quad (15)$$

$$\Delta\alpha_{\epsilon 1}/\Delta\gamma_\epsilon = p^2c^2/E(E+mc^2) = E_{min}/E \qquad \ldots \quad (16)$$

By $\Delta\alpha_{\epsilon 1}/\Delta\gamma_\epsilon$ we compare the rotation of spin orientation with the deflection of the beam.

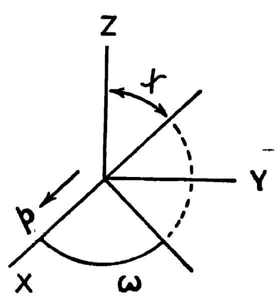

Fig. 1

From (16) we see that the spin orientation remains nearly constant in space for small kinetic energies. The corresponding value for transverse magnetic field is given by

$$\Delta\alpha_{\beta\perp} / \Delta\gamma_\beta = 1$$

where $\qquad \Delta\alpha_{\beta\perp} = -(e_\beta/pc)x. \qquad \Delta\gamma_\beta = -(e_\gamma/pc)x.$

The reason for this discrepancy is that the electric moment is v/c times less effective so far as rotation of spin direction in electric field is concerned than that of the magnetic moment and the deflection in magnetic field is v/c times less strong than that in electric field for the same magnitude of both electric and magnetic fields. In a similar manner we can treat the case of longitudinal electric field and also of the magnetic fields.

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